Dissipation at the two-loop level: Undressing the chiral condensate

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A simple and consistent real time analysis of the long-wavelength chiral condensate fields in the background of hard thermal modes is presented in the framework of the linear sigma model. Effective evolution equations are derived for the inhomogeneous condensate fields coupled to a heat bath. The multiple effects of the thermal background on the disoriented chiral condensate are studied using linear response theory. I determine the temperature dependence of the equilibrium condensate, and examine the modification of the sigma and pion dispersion relations as these mesons traverse a hot medium. I calculate the widths by determining the dissipative coefficients at nonzero temperature at one- and two-loop order with resummed meson masses. My results show that not only decay processes, but elastic scattering processes are significant at high temperatures, yielding to short relaxation times in the phase transition region. The relaxation times obtained are shorter than in previous estimates, making the observation of disoriented chiral condensate signals questionable. Throughout this work Goldstone's theorem is satisfied when chiral symmetry is spontaneously broken.

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I. INTRODUCTION

Nonequilibrium phenomena in many-body systems has received a great deal of attention during the last two decades. The theoretical interest keeps growing as new experiments are readily emerging in different research areas of physics, such as the chiral phase transition and quark-gluon plasma in ultrarelativistic heavy ion collisions, ultrafast spectroscopy in semiconductors, Bose-Einstein condensates, strongly correlated Fermi systems in condensed matter physics, and electroweak baryogenesis and inflation in early Universe cosmology. In the following I present a quantum field theoretical description of the dynamics of nuclear matter formed as a consequence of nuclear collisions at ultrarelativistic energies.

The existence of a deconfined, chirally symmetric phase of QCD was predicted long ago [1]. The interplay of the results obtained from the three major research approaches, lattice simulations, theoretical models and experiments at the BNL Relativistic Heavy Ion Collider (RHIC), lead us to expect that ''finding'' the quark-gluon plasma is within reach. PHENIX results [2] from the year-1 run of the RHIC suggest that the energy density achieved in Au+Au collisions already at $\sqrt{s}=130$ GeV is high enough to be favorable for the existence of free quarks and gluons. Early thermalization of the partonic medium [3] has been indicated by the elliptic flow analyses at STAR [4]. However, most probably thermal equilibrium is not maintained as matter rapidly expands and the temperature quickly drops through the QCD phase transition. Arguments that hint at nonequilibrium, even explosive [5] dynamics in heavy ion collisions, such as large fluctuations of the average transverse momentum, and almost equal sideward and outward Hanbury Brown–Twiss (HBT) radii, were reported in [6]. The dynamical evolution of such an out-of-equilibrium system is not yet understood. When it comes to possible approaches, lattice simulations unfortunately do not prove to be the way to go: a lattice can describe static situations in thermal equilibrium. Therefore, for learning about the dynamics of nonequilibrium systems, one has to reside to different theoretical approaches. Attempts are made using field theory [7–11], covariant kinetic theory [12], and recently also nonequilibrium fluid dynamics [13].

Until now no unambiguous observable that could serve as evidence for the phase transition has been identified. Based on the knowledge that a transition from ordered to disordered phase is accompanied by the formation of condensates, disoriented chiral condensates (DCC) have been suggested [14] as the signature of the chiral phase transition. This means that the matter formed in a heavy ion collision can relax into a vacuum state that is oriented differently than the normal ground state. Relaxing of DCCs to the correct ground state then happens through the emission of low momentum pions with an anomalous distribution in isospin space. Detecting fluctuations in the ratio of produced neutral pions compared to the charged ones can serve as a signal for the chiral symmetry restoring phase transition. Such DCC signals have not been observed at CERN-SPS energies [15]. The STAR detector at the RHIC searches for dynamical fluctuations on an event-by-event basis and may be therefore better suited for DCC searches. Other signals were proposed in context of DCCs: dileptons [16], and recently the anomaly in the Omega and anti-Omega abundances observed at the CERN Super Proton Synchrotron (SPS) [17].

The ability to detect DCCs depends on their lifetime. In the original work [14] DCC formation was proposed with the assumption of a perfect quench. This means that after the critical temperature is reached the long-wavelength modes of the chiral condensate decouple from the thermal modes and evolve according to zero temperature equations of motion. A more realistic discussion, accounting for the presence of a thermalized background was first presented in [18]. Because of the possible energy exchange between different degrees of freedom dissipation occurs, which in turn reduces the lifetime of the condensate. This problem has been addressed

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previously in [7–11,19,20]. Most of the previous calculations though focus on the evolution of the order parameter only [7,11]. Based on the observation that true pions have a finite width [21] the damping of pion condensates was considered [9,10], but only at one-loop level. The usual argument for neglecting two-loop contributions is that these are higher order in the coupling. However, there is an important reason for why two-loop contributions are not negligible: whereas decay processes can happen only under some kinematic conditions, scattering can always happen. Dissipation due to scattering has been evaluated for the scalar condensate in [11]. A first attempt to look at the lifetime of DCCs based on scattering processes was presented in [19].

In this paper I present a quantum field theoretical analysis of homogeneous and inhomogeneous chiral condensate field configurations out of thermal equilibrium that are coupled to a thermal bath. The framework is the linear sigma model, which has proved to describe fairly well the hadronic phase of two-flavor QCD. I treat the long wavelength chiral fields classically and account for the short wavelength fields perturbatively, but resumming certain diagrams. Such semiclassical description is acceptable since the occupation number of the low momentum modes is large. The effect of the heat bath on the condensate is contained within the deviations of the thermal field fluctuations from their equilibrium values. I identify these deviations as time-delayed responses of the hard thermal modes to the presence of the condensate, and evaluate them using linear response theory, which provides the evolution of observables in real physical time. The response functions renormalize the equations of motion, modifying the particle properties, and give rise to dissipation. Due to possible interaction and thus energy exchange between the soft and the hard modes decay channels open up and particles can scatter. These processes are responsible for the dissipation of the condensate. To the best of my knowledge, a consistent incorporation of relaxation processes at the one- and two-loop level has not been done before in this context. There is another important effect, that of the change in the velocity. I discuss this in [22].

It is important to determine the order of the chiral transition, as this influences the dynamical evolution of the system. Experimentally, large-acceptance detectors are now able to measure average as well as event-by-event observables, which in principle can distinguish between scenarios with a first order, a second order, or merely a crossover type of phase transition. QCD with two massless quarks possesses chiral symmetry, described by the $SU(2)_R \times SU(2)_L \simeq O(4)$ group. This continuous symmetry is spontaneously broken at low temperatures, resulting in the emergence of Goldstone bosons [23]. Based on universality arguments [24] the transition between the chiral symmetric and symmetry broken phases is of second order. Nature, however, supplies a different situation, with nonzero quark masses. Small quark masses explicitly break chiral symmetry, which then alters the nature of the phase transition. The second order transition becomes a smooth crossover, provided that the baryon density is zero. In case of a sufficiently large baryon chemical potential the transition is of first order, suggesting the existence of a tricritical point in the (μ_B, T) plane of the QCD phase diagram [25]. Year-1 RHIC results [2] suggest that at $\sqrt{s} = 130$ GeV the baryon density is approaching the low density limit. Expecting an even smaller baryon density at the maximum collision energy justifies us to work at finite temperature and zero baryon chemical potential.

The paper is structured as follows: In Sec. II I derive coarse-grained equations of motions for the long wavelength chiral condensate fields which are in in contact with a heat bath. In Sec. III the response of the thermal bath to the presence of the nonthermal condensate is evaluated. In Sec. IV I present the temperature-dependence of the equilibrium value of the condensate and that of the self-consistently calculated meson masses. It is important to emphasize that my analysis is performed with the satisfaction of Goldstone's theorem for all temperatures below the critical one. Also, the tachyon problem appearing in the mean field treatments is eliminated. In Sec. V I analyze the dissipation of long-wavelength sigma and pion fields due to decay and scattering processes. In Sec. VI the relaxation time of DCCs is determined. I summarize my results and give an outlook into further developments in the concluding Sec. VII.

II. DERIVING EQUATIONS OF MOTION

Within the linear sigma model framework the study of DCC formation and evolution is convenient, since the physical picture is rather transparent. The theory is formulated in terms of the chiral field $\Phi = (\sigma, \vec{\pi})$. The scalar sigma field σ describes the scalar quark condensate $\langle \bar{q}q \rangle$, which serves as order parameter. The pseudoscalar pion field $\vec{\pi} = (\pi^1, \pi^2, \ldots, \pi^{N-1})$ is directly related to the pseudoscalar condensate, $\langle \bar{q}\vec{\tau}\gamma_5 q \rangle$. The dynamics of the chiral condensate is completely determined by the evolution equations in space and time for the long-wavelength chiral fields. In the following I derive effective equations for these low-momentum fields in a hard-momentum background.

The Lagrangian of the linear $O(N)$ sigma model is

$$L(\Phi) = \frac{1}{2} \partial_\mu \Phi \partial^\mu \Phi - U(\Phi), \qquad (1)$$

where the usual choice for the potential is

$$U(\Phi) = \frac{\lambda}{4} (\Phi^2 - v_0^2)^2 + H\sigma. \qquad (2)$$

The explicit breaking of chiral symmetry is implemented through the H term that tilts the potential in the sigma direction. $H = f_\pi m_\pi^2$ and $v_0^2 = f_\pi^2 - m_\pi^2/\lambda$, where $f_\pi = 93$ MeV is the pion decay constant, λ is a positive dimensionless coupling constant, and $m_\pi = 138$ MeV is the zero temperature mass of the pion. The parameters of the Lagrangian are chosen such that for $H = 0$ chiral symmetry is spontaneously broken in the vacuum. The potential then resembles the bottom of a wine bottle with the zero temperature minimum at $\bar{\Phi} = (f_\pi, 0)$. The excitations in radial direction, the sigma mesons, have a mass $m_\sigma^2=2\lambda f_\pi^2$, and excitations along the azimuthal direction, the pions, are Goldstone bosons, with $m_\pi=0$.

The evolution of the fields in thermal equilibrium has been extensively studied [26,27] after the pioneering works by Linde [28] and Dolan and Jackiw [29]. The usual procedure is to express the fields as

$$\sigma(t,\vec{x})=v+\sigma_f(t,\vec{x}),$$

$$\pi^i(t,\vec{x})=\pi_f^i(t,\vec{x}),\quad i=1,\ldots,N-1. \tag{3}$$

The thermal average of the chiral field, $\overline{\Phi}=(v,0)$, is the order parameter chosen to lie along the sigma direction, $\langle\sigma\rangle_{eq}=v$ and $\langle\vec{\pi}\rangle_{eq}=0$. The fluctuations about the order parameter average to zero, $\langle\sigma_f\rangle_{eq}=\langle\vec{\pi}_f\rangle_{eq}=0$. Throughout this work we use the following notation: $\langle\mathcal{O}\rangle$ is the nonequilibrium ensemble average of an operator $\mathcal{O}$; $\langle\mathcal{O}\rangle_{eq}$ denotes the equilibrium, but interacting ensemble average; and $\langle\mathcal{O}\rangle_0$ is the free ensemble average.

For nonequilibrium thermal conditions, the idea is that instead of writing the fields as in Eq. (3), we separate them into their low and high frequency modes. This can be done, for example, by introducing a momentum cutoff Λ_c, as in [30]. Then, by integrating out the high frequency modes we obtain an effective theory for the low frequency, long wavelength modes. The fields can be separated as follows:

$$\sigma(x)=\tilde{\sigma}(x)+\sigma_f(x),$$

$$\pi^i(x)=\tilde{\pi}^i(x)+\pi_f^i(x),\quad i=1,\ldots,N-1. \tag{4}$$

$\tilde{\sigma}$ and $\tilde{\pi}^i$ are slowly varying condensate fields, representing low frequency modes with momentum $|\vec{k}|<\Lambda_c$. These soft modes are occupied by a large number of particles and may then be treated as classical fields. σ_f and π_f^i are high frequency, fast modes with $|\vec{k}|>\Lambda_c$. These hard modes, representing quantum and thermal fluctuations, constitute a heat bath. A choice of $\Lambda_c=0$ would mean that only homogeneous condensates, with $\vec{k}=0$ momentum are studied. Here I discuss condensates that can also be inhomogeneous.

The problem to be solved, then, is to describe the evolution of long wavelength classical fields that are embedded into a thermal bath. In my approach, these soft modes follow classical equations of motion, whereas the effect of the hard thermal modes is taken into account in a perturbative manner. I am well aware of the problems of a perturbative treatment, since the sigma model is a strongly coupled effective theory: With the choice of, for example, $m_\sigma\simeq 600$ MeV for the vacuum mass of the sigma meson the coupling constant is $\lambda\simeq 20$. However, I improve the model by resumming a certain class of diagrams in the perturbation series. I believe that even if the self-consistent solutions are approximate only, they still yield to qualitatively reliable results.

I average the Euler-Lagrange field equations over time and length scales that are short compared to the scales characterizing the change in the slow fields, but long relative to the scales of the quantum fluctuations. This is known as coarse-graining. The average of high frequency fluctuations is thus $\langle\sigma_f(x)\rangle=0$ and $\langle\vec{\pi}_f(x)\rangle=0$, while $\langle\tilde{\sigma}(x)\rangle=\tilde{\sigma}(x)$ and $\langle\tilde{\pi}^i(x)\rangle=\tilde{\pi}^i(x)$. It should be noted at this point that, compared to earlier works (for example [11]), I allow for a nonzero ensemble average not only along the sigma direction, but also in the pion directions. In other words, I allow the formation of disoriented chiral condensates. Also, cross correlations between fluctuations of different fields are nonzero, $\langle\varphi_i\varphi_j\rangle\neq 0$, and moreover, I consider nonzero cubic fluctuations of the form $\langle\varphi_i\varphi_j\varphi_k\rangle\neq 0$ which arise at the two-loop level. Furthermore, I separate the nonequilibrium condensate fields:

$$\tilde{\sigma}(x)=v+\sigma_s(x),$$

$$\tilde{\pi}^i(x)=\pi_s^i(x),\quad i=1,\ldots,N-1. \tag{5}$$

v is the equilibrium value of the chiral condensate chosen along the sigma direction and σ_s and π_s^i are slow fluctuations about equilibrium. The thermal equilibrium ensemble of the hard fluctuations is affected by the presence of the condensate. The full, nonequilibrium ensemble averages are basically two- and three-point functions of thermalized fields evaluated at the same space-time point. These can be written as the sum of the equilibrium ensemble average and a fluctuation about this:

$$\langle\sigma_f^2\rangle=\langle\sigma_f^2\rangle_{eq}+\delta\langle\sigma_f^2\rangle,$$

$$\langle\pi_f^{i\,2}\rangle=\langle\pi_f^{i\,2}\rangle_{eq}+\delta\langle\pi_f^{i\,2}\rangle,$$

$$\langle\sigma_f\pi_f^i\rangle=\delta\langle\sigma_f\pi_f^i\rangle,$$

$$\langle\pi_f^i\pi_f^j\rangle=\delta\langle\pi_f^i\pi_f^j\rangle,$$

$$\langle\sigma_f^3\rangle=\langle\sigma_f^3\rangle_{eq}+\delta\langle\sigma_f^3\rangle,$$

$$\langle\sigma_f\pi_f^{i\,2}\rangle=\langle\sigma_f\pi_f^{i\,2}\rangle_{eq}+\delta\langle\sigma_f\pi_f^{i\,2}\rangle,$$

$$\langle\pi_f^i\pi_f^{j\,2}\rangle=\delta\langle\pi_f^i\pi_f^{j\,2}\rangle,$$

$$\langle\sigma_f^2\pi_f^i\rangle=\delta\langle\sigma_f^2\pi_f^i\rangle. \tag{6}$$

The deviations in the fluctuations (of the general form $\delta\langle\varphi_i^n\varphi_j^m\rangle$) are the responses of the fast modes to the presence of slow σ_s and $\vec{\pi}_s$ background fields. These responses are proportional to the slow fields raised to some positive power, and so they vanish, as they should, in the absence of the background. Notice the absence of $\langle\sigma_f\pi_f^i\rangle_{eq}$ and $\langle\pi_f^i\pi_f^j\rangle_{eq}$. The reason for this is that the correlation functions in equilibrium, in the absence of a background, are diagonal. On account of the cubic couplings, however, one finds nonzero $\langle\sigma_f^3\rangle_{eq}$ and $\langle\sigma_f\pi_f^{i\,2}\rangle_{eq}$.

In equilibrium, at fixed temperature, $\sigma_s=0$ and $\vec{\pi}_s=0$ and the equilibrium condensate satisfies the following equation:

$$\lambda v^3+3\lambda v\langle\sigma_f^2\rangle_{eq}+\lambda v\sum_{i=1}^{N-1}\langle\pi_f^{i\,2}\rangle_{eq}+\lambda\langle\sigma_f^3\rangle_{eq}+\lambda\sum_{i=1}^{N-1}\langle\sigma_f\pi_f^{i\,2}\rangle_{eq}-\lambda v_0^2 v-H=0. \quad (7)$$

Since all components of the pion field are equivalent, one can write the sum as $\Sigma_{i=1}^{N-1}\langle\pi_f^{i\,2}\rangle_{eq}=(N-1)\langle\pi_f^2\rangle_{eq}$, where now π_f is one of the components. This is a convention that we adopt throughout the rest of the paper.

The resulting final equations describing the evolution of the condensate fields are a set of coupled nonlinear integro-differential equations. Let us assume only a small deviation from equilibrium, $|\sigma_s|,|\pi_s|\ll v$, and thus neglect terms that are higher order in σ_s and π_s. The linearized field equations read

$$\partial^2\sigma_s+M_\sigma^2\sigma_s+\lambda v[3\,\delta\langle\sigma_f^2\rangle+(N-1)\,\delta\langle\pi_f^2\rangle]+\lambda[\,\delta\langle\sigma_f^3\rangle+(N-1)\,\delta\langle\sigma_f\pi_f^2\rangle]=0,$$

$$M_\sigma^2=\lambda\left(2v^2-\frac{\langle\sigma_f^3\rangle_{eq}}{v}-(N-1)\frac{\langle\sigma_f\pi_f^2\rangle_{eq}}{v}\right)+\frac{H}{v}, \quad (8)$$

and

$$\partial^2\pi_s+M_\pi^2\pi_s+2\lambda v\,\delta\langle\sigma_f\pi_f\rangle+\lambda[\,\delta\langle\pi_f^3\rangle+\delta\langle\sigma_f^2\pi_f\rangle]=0,$$

$$M_\pi^2=\lambda\left(2\langle\pi_f^2\rangle_{eq}-2\langle\sigma_f^2\rangle_{eq}-\frac{\langle\sigma_f^3\rangle_{eq}}{v}-(N-1)\frac{\langle\sigma_f\pi_f^2\rangle_{eq}}{v}\right)+\frac{H}{v}. \quad (9)$$

I denote the effective masses of the sigma meson and the pion by M_σ and M_π, and their corresponding vacuum value by m_σ and m_π, respectively.

At one-loop order the effective equations become simpler:

$$\partial^2\sigma_s+M_\sigma^2\sigma_\sigma+\lambda v[3\,\delta\langle\sigma_f^2\rangle+(N-1)\,\delta\langle\pi_f^2\rangle]=0, \quad (10)$$

$$M_\sigma^2=2\lambda v^2+\frac{H}{v}, \quad (11)$$

and

$$\partial^2\pi_s+M_\pi^2\pi_s+2\lambda v\,\delta\langle\sigma_f\pi_f\rangle=0, \quad (12)$$

$$M_\pi^2=\frac{H}{v}+\tilde{m}_\pi^2=\frac{f_\pi}{v}m_\pi^2+\tilde{m}_\pi^2,\tilde{m}_\pi^2=2\lambda[\langle\pi_f^2\rangle_{eq}-\langle\sigma_f^2\rangle_{eq}], \quad (13)$$

and the equilibrium condensate satisfies

$$\lambda v^3+\lambda[3\langle\sigma_f^2\rangle_{eq}+(N-1)\langle\pi_f^2\rangle_{eq}]v-\lambda v_0^2 v-H=0. \quad (14)$$

III. NONEQUILIBRIUM FLUCTUATIONS

Linear response theory is a very convenient tool when one is interested in monitoring the effect of an external field applied to a system initially in thermal equilibrium. When this effect is small a linear approximation is feasible. A response function expresses the difference between the expectation value of an operator before and after an external perturbation has been turned on. The effects of the soft condensate fields on the thermal medium are thus described by expressions of the form

$$\delta\langle\sigma_f^n(x)\,\pi_f^m(x)\rangle=i\int_0^t dt'\int d^3x'\langle[H_{perturbed}(x'),\sigma_f^n(x)\,\pi_f^m(x)]\rangle_{eq}. \quad (15)$$

Accordingly, I evaluate the commutators of different powers of the field operators at two separate space-time points in the fully interacting, but unperturbed ensemble. Initial conditions are set by the assumption that in heavy-ion collisions the system reaches a state of approximate local thermal equilibrium [4], then it cools while expanding out of equilibrium and reaches the critical temperature. I define the initial time $t=0$ by when the critical temperature is reached. This implies $\sigma_s(0,\vec{x})=0$ and $\vec{\pi}_s(0,\vec{x})=0$. The respective powers n and m are identified from the expressions in Eqs. (8) and (9). Let me emphasize that these responses should be evaluated in the unperturbed, equilibrium ensemble, which does include all the interactions between the different modes.

The possible couplings between low and high frequency modes are determined by evaluating the potential U, with the fields separated into their slow and fast components as above. The resulting Hamiltonian contains positive powers of the slow fields. For small departures from equilibrium it is enough to keep the dominant linear terms only. Relaxing the assumption of only slightly out of equilibrium requires the inclusion of higher powers of the nonthermal fields. This should provide no difficulties, but is beyond the aim of the present paper. The relevant couplings are

$$H_{\sigma_s\sigma_f}=\lambda(3v^2-f_\pi^2)\sigma_s\sigma_f+3\lambda v\,\sigma_s\sigma_f^2+\lambda\,\sigma_s\sigma_f^3,$$

$$H_{\pi_s\pi_f}=\lambda(v^2-f_\pi^2)\,\pi_s\pi_f+\lambda\,\pi_s\pi_f^3,$$

$$H_{\sigma_s\pi_f}=\lambda v\,\sigma_s\pi_f^2,$$

$$H_{\pi_s\sigma_f\pi_f}=\lambda\,\pi_s\sigma_f^2\pi_f+2\lambda v\,\pi_s\sigma_f\pi_f,$$

$$H_{\sigma_s\sigma_f\pi_f}=\lambda\,\sigma_s\sigma_f\pi_f^2. \quad (16)$$

As before, $\pi_{s,f}$ refer to one of the $N-1$ components of the pion field.

The response functions obtained by inserting Eq. (16) in Eq. (15) are

$$\delta\langle\sigma_f^2(x)\rangle = 3i\lambda v\int_0^t dt'\int d^3x'\,\sigma_s(x') \times\langle[\sigma_f^2(x'),\sigma_f^2(x)]\rangle_{eq},$$

$$\delta\langle\pi_f^2(x)\rangle = i\lambda v\int_0^t dt'\int d^3x'\,\sigma_s(x') \times\langle[\pi_f^2(x'),\pi_f^2(x)]\rangle_{eq},$$

$$\delta\langle\sigma_f(x)\pi_f(x)\rangle = 2i\lambda v\int_0^t dt'\int d^3x'\,\pi_s(x') \times\langle[\sigma_f(x')\pi_f(x'),\sigma_f(x)\pi_f(x)]\rangle_{eq}. \tag{17}$$

The cubic functions are determined analogously, therefore we skip presenting their evaluation. It is clear that expressions (17) vanish in the absence of the nonthermal background, as they should.

The expectation values of the commutators are

$$\langle[\sigma_f^2(x'),\sigma_f^2(x)]\rangle_{eq} = 2(D_\sigma^<(x,x')^2 - D_\sigma^>(x,x')^2),$$

$$\langle[\pi_f^2(x'),\pi_f^2(x)]\rangle_{eq} = 2(D_\pi^<(x,x')^2 - D_\pi^>(x,x')^2),$$

$$\langle[\sigma_f(x')\pi_f(x'),\sigma_f(x)\pi_f(x)]\rangle_{eq} = D_\sigma^<(x,x')D_\pi^<(x,x') - D_\sigma^>(x,x')D_\pi^>(x,x'),$$

where $x=(t,\vec{x})$ and $x'=(t',\vec{x}')$ are four-vectors in coordinate space. Keeping in mind that $t'<t$, where t is the time elapsed after switching on the perturbation, and t' is the time-variable that has its values in the $[0,t]$ interval, the following notation has been introduced:

$$D_\sigma^>(x,x') \equiv \langle\sigma_f(x)\sigma_f(x')\rangle_{eq},$$

$$D_\pi^>(x,x') \equiv \langle\pi_f(x)\pi_f(x')\rangle_{eq}$$

and

$$D_\sigma^<(x,x') \equiv \langle\sigma_f(x')\sigma_f(x)\rangle_{eq},$$

$$D_\pi^<(x,x') \equiv \langle\pi_f(x')\pi_f(x)\rangle_{eq}. \tag{18}$$

The functions $D^>$ and $D^<$ define the spectral function

$$\rho(k) = D^>(k) - D^<(k), \tag{19}$$

which determines the real time propagator

$$D(x,x') = \int\frac{d^4k}{(2\pi)^4}e^{-ik(x-x')}D(k) = \int\frac{d^4k}{(2\pi)^4}e^{-ik(x-x')}(\Theta(t-t')+f(k^0))\rho(k). \tag{20}$$

Here $k=(k^0,\vec{k})$ is the four-momentum, $f(k^0)=(e^{k^0\beta}-1)^{-1}$ is the Bose-Einstein distribution, and $\beta=T^{-1}$. The response functions are then

$$\delta\langle\sigma_f^2(x)\rangle = -i6\lambda v\int d^4x'\,\sigma_s(x')\int\frac{d^4p}{(2\pi)^4} \times\int\frac{d^4q}{(2\pi)^4}e^{-i(p+q)(x-x')}\rho_\sigma(p)\rho_\sigma(q) \times[1+f(p^0)+f(q^0)],$$

$$\delta\langle\pi_f^2(x)\rangle = -i2\lambda v\int d^4x'\,\sigma_s(x')\int\frac{d^4p}{(2\pi)^4} \times\int\frac{d^4q}{(2\pi)^4}e^{-i(p+q)(x-x')} \times\rho_\pi(p)\rho_\pi(q)[1+f(p^0)+f(q^0)],$$

$$\delta\langle\sigma_f(x)\pi_f(x)\rangle = -i2\lambda v\int d^4x'\,\pi_s(x')\int\frac{d^4p}{(2\pi)^4} \times\int\frac{d^4q}{(2\pi)^4}e^{-i(p+q)(x-x')} \times\rho_\sigma(p)\rho_\pi(q)[1+f(p^0)+f(q^0)]. \tag{21}$$

IV. THE ORDER PARAMETER AND MESON MASSES

To analyze the phase transition I look at the temperature dependence of the equilibrium condensate which is the order parameter of my model. The behavior of v is determined by solving Eq. (7). It is educational to look at the theory in the exact chiral limit, $H=0$, first. In this case Eq. (7) for the order parameter has two solutions:

$$v=0,\quad T>T_c,$$

and

$$v^2 = f_\pi^2 - 3\langle\sigma_f^2\rangle_{eq} - (N-1)\langle\pi_f^2\rangle_{eq} - \frac{\langle\sigma_f^3\rangle_{eq}}{v} - (N-1)\frac{\langle\sigma_f\pi_f^2\rangle_{eq}}{v},\quad T<T_c. \tag{22}$$

The first solution shows, as is expected, that the chiral condensate does not exist above a critical temperature. The second solution represents the low temperature, symmetry broken phase. The two solutions are separated by a second order phase transition.

The order parameter depends on the equilibrium field fluctuations, which to first approximation in either perturbative expansion in λ [31] or $1/N$-expansion [27], are

$$\langle \varphi^2 \rangle_{eq} = \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E}[1+2f(E)], \tag{23}$$

where $\varphi = \sigma_f, \pi_f$, and f is the Bose-Einstein distribution function of the mesons with energy $E=\sqrt{p^2+m^2}$. Cubic fluctuation terms that enter in the expression for the equilibrium condensate, $\langle \sigma_f^3 \rangle_{eq}/v$ and $\langle \sigma_f \pi_f^2 \rangle_{eq}/v$, are nonzero on account of the possible cubic couplings. However, already the leading order term of the expansion is higher order in the coupling than Eq. (23), so we can drop these to first order.

Let me discuss some important observations regarding the fluctuations (23). First, keeping only the leading order term means that the masses are the bare zero temperature masses. However, as indicated by Eqs. (11) and (13), the masses themselves are given in terms of the equilibrium condensate and as such, they are temperature dependent. Therefore the temperature dependence of the order parameter should be determined by performing a self-consistent evaluation. Here I resum tadpole diagrams.

Second, the first term is the vacuum contribution, while the second term is due to finite temperature effects. The zero temperature part is divergent in the ultraviolet limit. This divergence can and should be removed using vacuum renormalization techniques. The finite temperature does not introduce any extra divergence since it is regularized by the distribution function. One can say that the very short distance behavior of the theory is not affected by finite temperature [31]. Therefore $T=0$ renormalization is enough to obtain finite results. The usual approach then is to neglect the zero temperature contributions when focusing on the physics at finite temperatures. At this point, it is worth mentioning that using a self-consistent approximation makes the usual renormalization procedure difficult, as discussed in [32]. The argument, according to which the renormalized divergent term can be neglected, is really correct only when the mass is the bare mass. A self-consistent calculation involves the temperature-dependent mass, leading to the temperature-dependence of the divergent term. Renormalization thus results in temperature-dependent renormalization constants, and these should not be ignored. However, such a treatment is beyond the scope of this paper, and in what follows, I am going to ignore the divergent term, while still being aware of this approximation.

Third, the momentum integration has a lower limit, Λ_c, due to the restriction of the thermal population to hard momenta, $|\vec{p}|>\Lambda_c$. When $\Lambda_c=0$, the condensate contains only zero momentum modes, meaning that the classical field configurations are homogeneous. In reality nonzero but small momenta can be part of the condensate. Then one talks about inhomogeneous condensate.

A. Goldstone modes

In the theory with spontaneously broken chiral symmetry the tree level mass of the pion is zero in the broken phase, $m_\pi=0$. Goldstone's theorem [23] requires that this remains zero at every order in perturbation theory. The first glimpse at Eq. (9) [or Eq. (13)] for the pion condensate shows a mass term. At one-loop order

$$\tilde{m}_\pi^2 = 2\lambda[\langle \pi_f^2 \rangle_{eq} - \langle \sigma_f^2 \rangle_{eq}], \tag{24}$$

which is zero only at (i) zero temperature, where the thermal fluctuations themselves vanish, or (ii) when the masses of the pion and the sigma are equal, which is expected at the critical temperature, or (iii) in the high temperature limit, when the meson masses can be neglected. In the following, I present a simple and clear way to prove that this violation of Goldstone's theorem is only apparent.

The pion mass (24) includes one-loop tadpole contributions. There is another one-loop diagram that contributes to order λ, "built" out of two 3-vertices, known as the sunset diagram. This diagram is incorporated in the equation of motion through the response function $\delta\langle \pi_f \sigma_f \rangle$ given by Eq. (21). The function is of the order of $\lambda v \sim \lambda^{1/2}$, and there is an overall factor of $2\lambda v \sim \lambda^{1/2}$ in front of it in the effective equation. Therefore to get the contribution of order λ, the expectation value in Eq. (21) can be evaluated at the lowest order. This means that I can replace the interacting ensemble average $\langle \cdots \rangle_{eq}$ by the free ensemble average $\langle \cdots \rangle_0$. This in turn is equivalent to inserting free spectral functions,

$$\rho_{\text{free}}(p) = 2\pi\epsilon(p^0)\delta(p^2-m^2), \tag{25}$$

into the expressions of Eq. (21). After evaluating the frequency integrals I find

$$\delta\langle \sigma_f \pi_f \rangle = i2\lambda M_\sigma^2 \int d^4x' \pi_s(x') \int \frac{d^3p}{(2\pi)^3} \times \int \frac{d^3q}{(2\pi)^3} e^{i(\vec{p}+\vec{q})(\vec{x}-\vec{x}')} F(\vec{p},\vec{q},t'), \tag{26}$$

with

$$F(\vec{p},\vec{q},t') = \frac{1}{4E_\sigma E_\pi}[(1+f_\sigma+f_\pi)(e^{i(E_\sigma+E_\pi)(t-t')} - e^{-i(E_\sigma+E_\pi)(t-t')}) + (f_\pi - f_\sigma) \times (e^{i(E_\sigma-E_\pi)(t-t')} - e^{-i(E_\sigma-E_\pi)(t-t')})].$$

Since, the deviation from equilibrium is assumed to be small, I Taylor expand $\pi_s(x')$ about its equilibrium value. The first term of the expansion, linear in $\pi_s(x)$, gives the contribution to the mass. The total pion mass is then

$$M_\pi^2 = \tilde{m}_\pi^2 + a_1, \tag{27}$$

where

$$
\begin{aligned}
a_1 &= i2\lambda M_\sigma^2 \int \frac{d^3p}{(2\pi)^3}\int_0^t dt' F(\vec{p},t') \\
&= i2\lambda M_\sigma^2 \int \frac{d^3p}{(2\pi)^3}\frac{1}{4E_\sigma E_\pi} \\
&\quad \times \left[\frac{2E_\sigma(1+2f_\pi)-2E_\pi(1+2f_\sigma)}{i(E_\pi^2-E_\sigma^2)}\right] \\
&= 2\lambda \int \frac{d^3p}{(2\pi)^3}\left[\frac{1}{2E_\sigma}(1+2f_\sigma)-\frac{1}{2E_\pi}(1+2f_\pi)\right],
\end{aligned}
\tag{28}
$$

and $E_\sigma=\sqrt{\vec{p}^2+M_\sigma^2}$ and $E_\pi=|\vec{p}|$. Summing up Eqs. (28) and (24) in expression (27), the true pion mass yields

$$M_\pi=0, \tag{29}$$

thus proving that with proper inclusion of diagrams the pions stay Goldstone bosons at one-loop level at all temperatures. Another simple and straightforward proof in frequency-momentum space is presented in [22].

B. Numerical results

In the exact chiral limit the self-consistent solution of the gap equation

$$v^2=f_\pi^2-3\langle\sigma_f^2\rangle_{eq}-(N-1)\langle\pi_f^2\rangle_{eq}, \tag{30}$$

with the sigma and pion field fluctuations

$$\langle\sigma_f^2\rangle=\frac{1}{2\pi^2}\int_{\Lambda_c}^\infty dp\ \frac{p^2}{E_\sigma}\frac{1}{e^{E_\sigma/T}-1}, \tag{31}$$

and

$$\langle\pi_f^2\rangle=\frac{1}{2\pi^2}\int_{\Lambda_c}^\infty dp\ \frac{p^2}{E_\pi}\frac{1}{e^{E_\pi/T}-1}\rightarrow\frac{T^2}{12},$$
$$\text{for}\ \ M_\pi\rightarrow0,\quad \Lambda_c\rightarrow0, \tag{32}$$

evaluated with $E_\sigma=\sqrt{p^2+M_\sigma^2}$, where

$$M_\sigma^2=2\lambda v^2, \tag{33}$$

is presented in Fig. 1 for different values of the momentum separation scale Λ_c. In the numerical analysis $N=4$ and the coupling constant was chosen to be $\lambda=18$, corresponding to a vacuum sigma mass of about $m_\sigma^2=2\lambda f_\pi^2=(558\ \text{MeV})^2$. The phase transition temperature, defined by the vanishing of the condensate, is about $T_c\simeq130$ MeV for homogeneous classical field configurations, $\Lambda_c=0$. With the increase of the scale Λ_c the phase-space available for hard modes is decreased, requiring higher temperatures for the fluctuations to completely dissolve the condensate. For $\Lambda_c=120$ MeV the temperature above which fluctuations are much too large to allow the formation of any condensate is about $T_c \simeq165$ MeV, which is in good agreement with lattice data [33]. Figure 1 shows that the meson mass is positive-definite at all temperatures, eliminating the tachyon problem present in the mean field approximations [31].

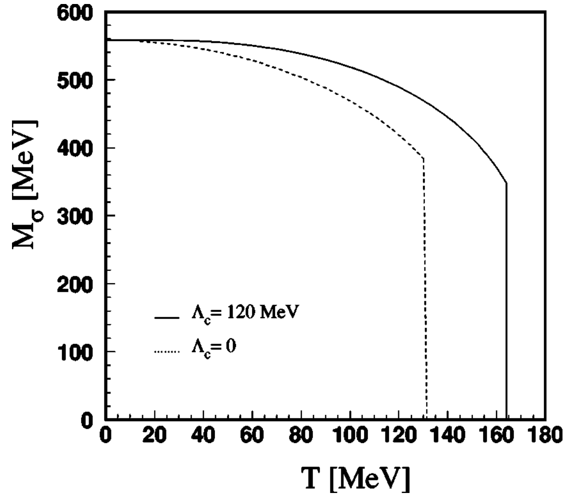

FIG. 1. Temperature dependence of the sigma mass for different momentum separation scales in the chiral limit.

The theory with exact chiral symmetry is known to have a second order phase transition based on universality arguments [24]. This has been confirmed within the mean field approximation [27]. Figure 1, however, shows a discontinuous behavior of the order parameter at T_c. Such a jump is characteristic to first-order phase transitions. Incorporating the effect of thermal fluctuations in a self-consistent way renders the transition first order. Such a behavior has been discussed many years ago also by Baym and Grinstein [26]. In Fig. 2 I present how the discontinuity decreases with decreasing coupling constant. For a weak enough coupling the continuous second-order transition is recovered. This can be understood in the following way: in order to assure the finiteness of the $O(N)$-theory in the large-N limit the coupling constant should be written as λ/N [26] in the self-consistent gap equation (33)–(30):

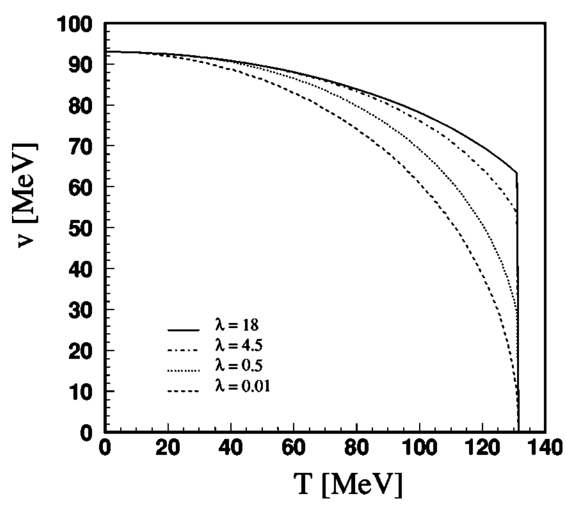

FIG. 2. Temperature dependence of the equilibrium condensate for different coupling constants in the chiral limit.

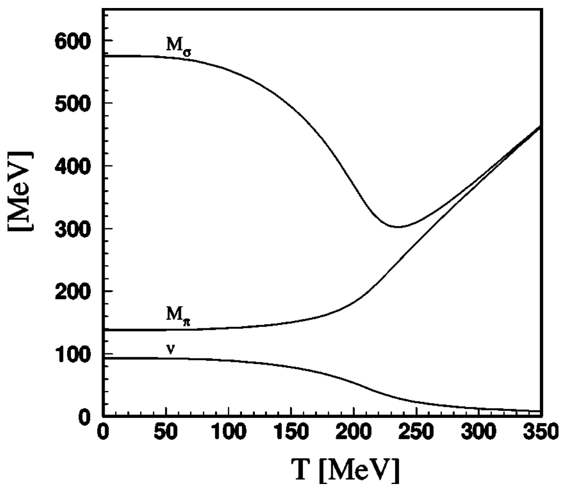

FIG. 3. Temperature dependence of the resummed meson masses and of the equilibrium condensate in the $O(4)$ model with explicitly broken symmetry.

$$M_\sigma^2 = m_\sigma^2 - 2\frac{\lambda}{N}[3\langle\sigma_f^2\rangle_{eq} + (N-1)\langle\pi_f^2\rangle_{eq}]. \tag{34}$$

For $N\to\infty$ the contribution from the sigma field fluctuation disappears. The condensate is then

$$v^2 = f_\pi^2 - \frac{N-1}{N}\langle\pi_f^2\rangle_{eq} \simeq f_\pi^2 - \frac{T^2}{12} \tag{35}$$

clearly showing a continuous decrease of the order parameter with increasing temperature. In the $O(4)$ model decreasing λ by hand is equivalent to going to the large N limit in the $O(N)$ model. For our model parameters I have determined a large $N_{\rm critical}\simeq 1800$ at which the transition is second order. For this $\lambda = 18$ was held fixed. This result is equivalent to having $\lambda_{\rm critical}\simeq 0.01$ and $N=4$.

Numerically determined self-consistent solutions for the condensate (14) and the meson masses (11) and (13) in the more realistic theory with explicitly broken symmetry are displayed in Fig. 3 and show a qualitatively different behavior. There is no phase transition in the textbook sense. The equilibrium condensate monotonically decreases with increasing temperature. A crossover region can be defined where the sigma and pion masses start to approach degeneracy. The minimum of the sigma mass is at $T\simeq 235$ MeV. Different values for Λ_c do not introduce a significant effect in evaluating the meson masses.

V. DISSIPATION OF THE CHIRAL CONDENSATE

Dissipation of the condensate occurs because energy can be transferred between the condensate and the heat bath through the interactions of soft and hard degrees of freedom. Formally, in our model, the damping of different modes is determined from the response functions. Most of the previous studies on this topic have been done at the one-loop level. The two-loop level scattering processes have been ignored, based on the argument that these are higher order in the coupling constant than are the decay and absorbtion processes. However, as I show in the following, two-loop processes can dominate due to the available large phase space.

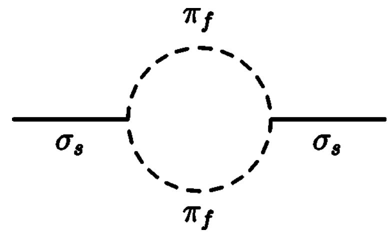

FIG. 4. One-loop self-energy contribution to the soft σ_s with coupling to the hard π_f through $\lambda v\sigma_s\pi_f^2$.

The effective equations of motion for long-wavelength meson fields (10) and (12) have the general form

$$\partial^2\phi_s(x) + M^2\phi_s(x) + F(x) = 0, \tag{36}$$

where $\phi_s = \sigma_s$ or π_s and

$$F(x) = \int d^4x'\,\phi_s(x')\Pi(x-x'). \tag{37}$$

In frequency-momentum space this reads as

$$-k^2 + M^2 + \Pi(k) = 0. \tag{38}$$

The function $\Pi(k)$ is given by

$$\Pi(k) = -ig^2\int d^4x'\int\frac{d^4p}{(2\pi)^4}\int\frac{d^4q}{(2\pi)^4}e^{i(k-p-q)(x-x')} \times\rho_1(p)\rho_2(q)[1+f(p^0)+f(q^0)] \tag{39}$$

and is identified as the self-energy. Here g is the corresponding coupling, and the indices 1 and 2 refer to either of the hard modes, σ_f and π_f, respectively, and $k=(k^0,\vec{k})$ is the four-momentum of the soft sigma meson or pion. The frequency has a real and an imaginary part, $k^0=\omega - i\Gamma$, provided $\vec{k}$ is real. The real part of the self-energy participates in the dispersion relation

$$\omega^2 = \vec{k}^2 + M^2 + {\rm Re}\,\Pi(\omega,\vec{k}), \tag{40}$$

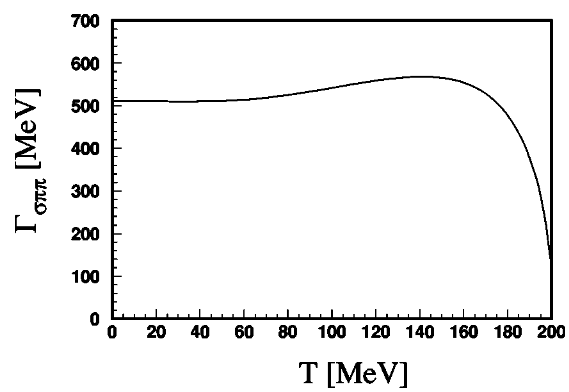

FIG. 5. Temperature dependence of the sigma damping rate at one-loop order in the sigma rest frame. Calculations were done with resummed meson masses.

and with the usual assumption of weak damping, $\Gamma \ll \omega$, the imaginary part of the self-energy completely determines the damping of excitations:

$$\Gamma = -\frac{\mathrm{Im}\,\Pi(\omega,\vec{k})}{2\omega}. \tag{41}$$

Γ is the rate at which out-of-equilibrium meson modes with energy ω and momentum $\vec{k}$ approach equilibrium. This rate is determined by physical processes which can be identified from the imaginary part of the self-energy [34]. One should be aware that Eq. (41) describes the rate of decay of the amplitude of the wave, $\exp(-\Gamma t)$. The loss rate for the number density is $\exp(-2\Gamma t)$.

A. Dissipation at one-loop order

There are several diagrams contributing to the self-energy at one-loop order. Tadpoles are real and they only modify the mass, as I discussed before. Dissipative effects come from nonlocal diagrams. At order λ these are determined from the response functions through Eq. (39) with the insertion of the free spectral functions,

$$\Pi(k) = g^2 \int \frac{d^3p}{(2\pi)^3} \frac{1}{4E_1E_2}\left[(1+f_1+f_2)\left(\frac{1}{\omega-E_1-E_2+i\epsilon} - \frac{1}{\omega+E_1+E_2+i\epsilon}\right) + (f_2-f_1)\left(\frac{1}{\omega-E_1+E_2+i\epsilon} - \frac{1}{\omega+E_1-E_2+i\epsilon}\right)\right],$$

with $E_1=\sqrt{m_1^2+(\vec{p}+\vec{k})^2}$ and $E_1=\sqrt{m_2^2+\vec{p}^2}$. The indices 1 and 2 refer again to either of the fast sigma and pion. It is important to observe that the above expression coincides with the self-energy calculated directly from the nonlocal one-loop diagram [34]. For positive energies of the mesons $\omega \geq 0$ the contributions to the imaginary part of the self-energy are

$$\mathrm{Im}\,\Pi(\omega,\vec{k}) = -\pi g^2 \int \frac{d^3p}{(2\pi)^3} \frac{1}{4E_1E_2} \times [(1+f_1+f_2)\,\delta(\omega-E_1-E_2) + (f_2-f_1)\,\delta(\omega-E_1+E_2)]. \tag{42}$$

Because the heat bath singles out a preferred reference frame we keep ω and $\vec{k}$ independent. The dynamics of the decay is determined by the on-shell processes that are allowed.

1. Sigma meson decay

Contributions to the imaginary part of the sigma meson self-energy come from the diagram presented in Fig. 4. Kinematically, the decay of a soft sigma meson into hard thermal pions, $\sigma_s \to \pi_f \pi_f$, and the inverse, recombination process is allowed, provided that $\omega^2-k^2 \geq 4m_\pi^2$:

$$\mathrm{Im}\,\Pi(\omega,\vec{k}) = -\frac{g^2}{16\pi}\left[\sqrt{1-\frac{4m_\pi^2}{\omega^2-k^2}} + 2\frac{T}{k}\log\left(\frac{1-e^{-(\omega_+/T)}}{1-e^{-(\omega_-T)}}\right)\right], \tag{43}$$

where

$$\omega_\pm = \frac{\omega}{2} \pm \frac{k}{2}\sqrt{1-\frac{4m_\pi^2}{\omega^2-k^2}}.$$

The decay and formation processes are obtained by setting $\omega^2-k^2=m_\sigma^2$:

$$\mathrm{Im}\,\Pi(m_\sigma) = -\frac{\lambda^2 v^2}{16\pi}\left[\sqrt{1-\frac{4m_\pi^2}{m_\sigma^2}} + 2\frac{T}{k} \times \log\left(\frac{1-\exp\left(-\frac{\omega}{2T}-\frac{k}{2T}\sqrt{1-\frac{4m_\pi^2}{m_\sigma^2}}\right)}{1-\exp\left(-\frac{\omega}{2T}+\frac{k}{2T}\sqrt{1-\frac{4m_\pi^2}{m_\sigma^2}}\right)}\right)\right]. \tag{44}$$

The rate at which soft, $k \ll T$, sigmas of energy ω disappear from the condensate due to their decay into thermal pions is

$$\Gamma_{\sigma\pi\pi}(\omega) = \frac{(N-1)}{16\pi}\lambda\frac{m_\sigma^2-m_\pi^2}{\omega}\sqrt{1-\frac{4m_\pi^2}{m_\sigma^2}}\coth\left(\frac{\omega}{4T}\right). \tag{45}$$

The temperature dependence of the sigma damping rate (45) in the rest frame of the sigma, $\omega=m_\sigma$, is shown in Fig. 5. The calculations were done with the resummed meson masses, M_σ and M_π. Figure 5 shows that even at zero temperature there is a finite damping, so the sigma meson can decay into two pions in the vacuum. At $T=0$, for our model parameters the damping rate is about $\Gamma_{\sigma\pi\pi}=510$ MeV,

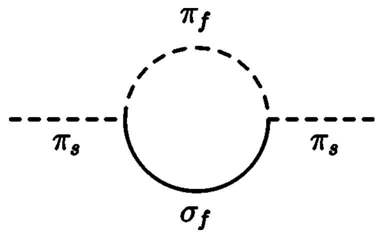

FIG. 6. One-loop self-energy contribution to π_s with coupling to σ_s and π_s through $2\lambda v\pi_s\sigma_f\pi_f$.

which is of the order of the mass of the sigma. The width of the sigma resonance is very broad, in other words the sigma meson is overdamped. Figure 5 shows that the damping is increasing with T, and is followed by a quite drastic decrease starting from about $T=150$ MeV. This temperature corresponds to the temperature where the sigma mass begins to drop significantly (see Fig. 3). It is then natural to expect a decrease of the sigma decay rate into pions that have masses approaching that of the sigma. Above $T\simeq 200$ MeV the threshold condition for the decay of an on-shell sigma meson, $M_\sigma \geqslant 2M_\pi$, is not fulfilled anymore. Therefore there will be no contribution from decay to the sigma width in the kinematically suppressed region.

2. Pion damping

At one-loop order there is only one diagram contributing to the pion self-energy. This diagram is shown in Fig. 6. At this order, dissipation of the pion condensate can occur provided the energy and momentum of the soft pion satisfies the kinematic condition $\omega^2-k^2\leqslant(m_\sigma-m_\pi)^2$. Then the transformation of a pion into a sigma when propagating through a thermal medium can happen. Basically, a soft pion from the condensate annihilates with a hard thermal pion producing a hard thermal sigma meson, $\pi_s\pi_f\to\sigma_f$. The inverse process is the decay of a hard thermal sigma meson into a soft and a hard pion, $\sigma_f\to\pi_s\pi_f$. The net rate of dissipation is

$$\Gamma_{\pi\pi\sigma}(\omega,\vec{k})=\frac{1}{8\pi}\lambda\frac{T(m_\sigma^2-m_\pi^2)}{k\omega}\left[\log\left(\frac{e^{\omega_+/T}-1}{e^{\omega_-/T}-1}\right)+\log\left(\frac{e^{(\omega_++\omega)/T}-1}{e^{(\omega_-+\omega)/T}-1}\right)\right], \tag{46}$$

where

$$\omega_\pm=\sqrt{p_\pm^2+m_\pi^2},$$

$$p_\pm=\pm\frac{k}{2}\frac{m_\sigma^2-2m_\pi^2}{m_\pi^2}+\frac{\omega}{2}\frac{m_\sigma^2}{m_\pi^2}\sqrt{1-\frac{4m_\pi^2}{m_\sigma^2}}.$$

In the rest frame of the massive pion the expression for the damping is simplified to

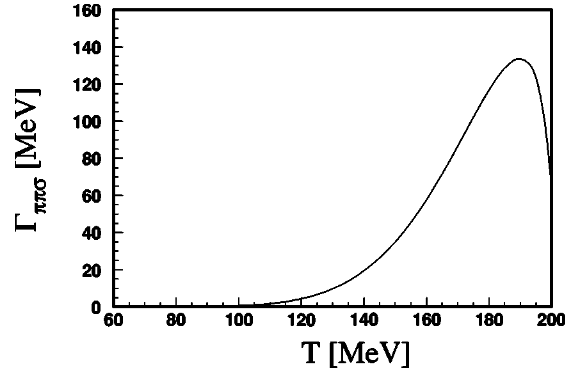

FIG. 7. Temperature dependence of the pion damping rate at one-loop order in the pion rest frame. Calculations were done with resummed meson masses.

$$\Gamma_{\pi\pi\sigma}(m_\pi)=\frac{\lambda}{16\pi}\frac{m_\sigma^2(m_\sigma^2-m_\pi^2)}{m_\pi^3}\sqrt{1-\frac{4m_\pi^2}{m_\sigma^2}}\times\frac{1-e^{-(m_\pi/T)}}{(e^{(m_\sigma^2-2m_\pi^2)/2m_\pi T}-1)(1-e^{-(m_\sigma^2/2m_\pi T)})}. \tag{47}$$

Figure 7 shows the temperature dependence of the damping rate of massive pions at one-loop order (47) calculated in the pion's rest frame using the resummed meson masses, M_σ and M_π. Note that at zero temperature the dissipation is zero. This makes sense because the transformation of pions into sigmas is due to their annihilation with a hard thermal pion in the medium, and so this is exclusively a finite temperature process. At low temperatures the phase space available for this pion transformation process is suppressed by the large sigma mass. As the temperature increases and the sigma mass is dropping the width of the pions is increasing. Figure 7 shows that the damping can get quite strong. At $T\simeq 170$ MeV, for example, when the pion mass is about $m_\pi=158$ MeV the damping is $\Gamma_{\pi\pi\sigma}\simeq 87.0$ MeV. Thus at this temperature the width of the pion is about 55% of its energy. This result makes us question whether the pion is a quasiparticle in this temperature region and needs further investigations. At temperatures around 200 MeV the kinematic condition for an on-shell pion, $M_\sigma\geqslant 2M_\pi$, is not satisfied, prohibiting the transformation of a pion into a sigma while passing through a hot medium.

The damping in terms of the pion momenta is presented in Fig. 8 for different temperatures. Results using the dispersion relation $\omega^2=k^2+M_\pi^2$ are displayed. Noticeable damping occurs above $T=100$ MeV and increases with T. At about $T=160$ MeV all modes are equally damped. In other words, the width of the pion is independent of its momentum. This width is increasing with T and it can be as great as 30% of the energy. At even higher temperatures the damping of the zero momentum modes is the strongest.

B. Dissipation at two-loop order

Instead of the tedious evaluation of two-loop linear response functions, I present a direct determination of damping

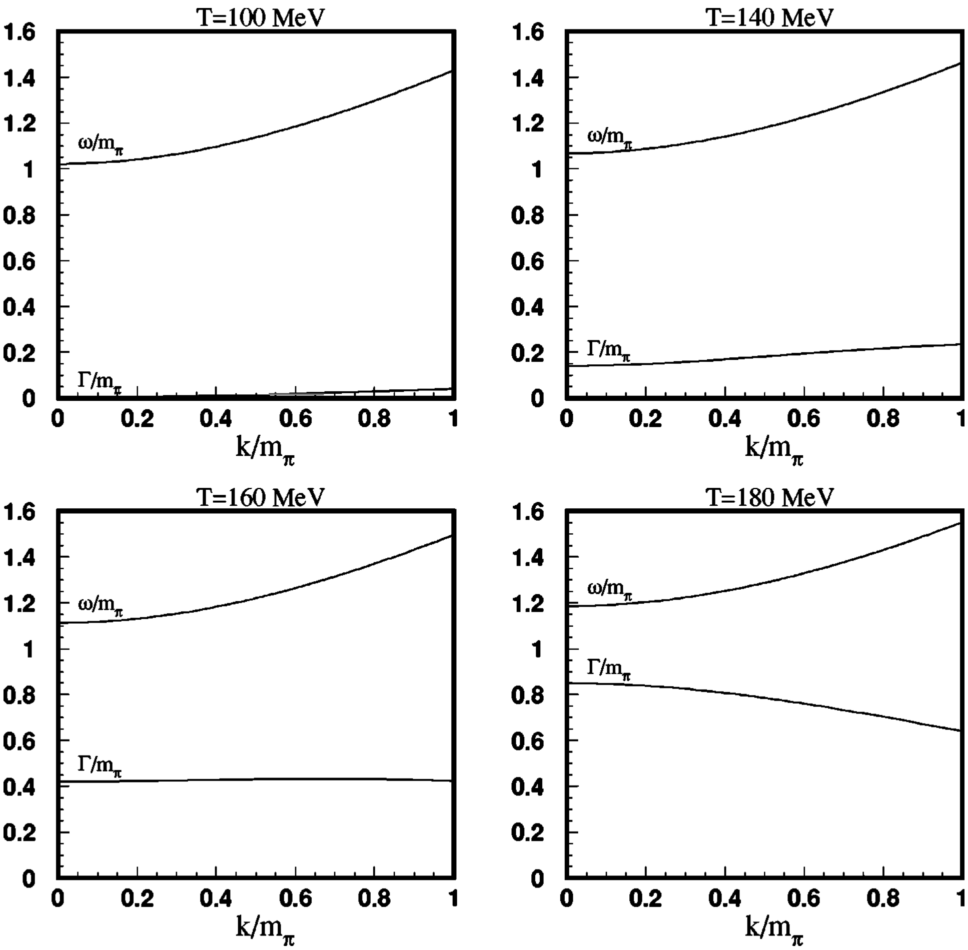

FIG. 8. Momentum dependence of the pion energy and width at different temperatures calculated with resummed meson masses and with the dispersion relation $\omega^2 = k^2 + M_\pi^2$. All quantities are normalized to the vacuum pion mass.

rates from the physical processes responsible. Two-loop contributions correspond to two-particle scatterings with amplitudes evaluated at tree level. Similar to the one-loop calculations I evaluate the imaginary parts of the two-loop self-energies and inserting these into Eq. (41) results in the damping rate due to scattering.

The general form of the self-energy of a particle of mass m_a, propagating with four-momentum $k = (\omega, \vec{k})$ through a medium in thermal equilibrium, is given by [35]

$$\Pi_{ab}(k) = \int \frac{d^3p}{(2\pi)^3 2E} f(E) \mathcal{M}(s). \tag{48}$$

Here $\mathcal{M}$ is the transition amplitude for the scattering process $ab \to ab$. The thermodynamical weight $f(E)$ is the Bose distribution of thermal mesons of mass m_b and four-momentum $p = (E, \vec{p})$. In terms of the forward scattering amplitude $\mathcal{M}(s) = -8\pi\sqrt{s} f_{cm}(s)$, where $s = (p+k)^2$, and the imaginary part follows:

$$\mathrm{Im}\,\Pi_{ab}(k) = -\int \frac{d^3p}{(2\pi)^3} f(E) \sqrt{s} \frac{q_{cm}}{E} \sigma_{\mathrm{total}}(s). \tag{49}$$

To obtain this equality I applied the standard form of the optical theorem that relates the imaginary part of the forward scattering amplitude and the total cross section [36]:

$$\mathrm{Im}\, f_{cm}(s) = \frac{q_{cm}}{4\pi} \sigma_{\mathrm{total}}(s). \tag{50}$$

Here I consider scatterings involving massive particles only. Then

$$\mathrm{Im}\,\Pi_{ab}(\omega) = -\frac{1}{8\pi^2} \int_{-1}^{1} d\cos\theta \times \int_{\Lambda_c}^{\infty} dp \frac{p^2}{E} f(E) \sigma_{ab}(E) \times \sqrt{(s - m_b^2 + m_a^2)^2 - 4 s m_a^2},$$

with $s = m_a^2 + m_b^2 + 2E\omega - 2pk\cos\theta$, where θ is the angle between $\vec{k}$ and $\vec{p}$. The dispersion relation of the hard thermal modes is $E = \sqrt{p^2 + m_b^2}$ and of the mesons inside the condensate $\omega = \sqrt{k^2 + m_a^2}$. It is convenient to work in the rest frame of meson a, where

$$\mathrm{Im}\,\Pi_{ab}(\omega = m_a, \vec{k} = 0) = -\frac{m_a}{2\pi^2} \int_{m_b}^{\infty} dE (E^2 - m_b^2) f(E) \sigma_{ab}(E). \tag{51}$$

The cross section for a scattering $ab \to ab$ is given by

$$\sigma_{ab}=\frac{1}{S!}\int\left(\frac{d\sigma_{ab}}{d\Omega}\right)_{cm}d\Omega \tag{52}$$

where

$$\left(\frac{d\sigma_{ab}}{d\Omega}\right)_{cm}=\frac{1}{64\pi^2 s}|\mathcal{M}|^2. \tag{53}$$

The symmetry factor $1/S!$ is due to the number S of identical particles in the final state. It is clear that knowing the amplitude $\mathcal{M}$ of a process readily results in the dissipation rate due to that process.

1. Sigma scattering

There are two mechanisms that contribute to the removal or addition of a sigma meson to the condensate: elastic scattering of a hard thermal sigma or of a hard thermal pion off a sigma from the condensate. To first order in the coupling λ there are four diagrams contributing to the process in which a thermal sigma meson knocks out a low momentum sigma from the condensate. The transition amplitude, the sum of contributions from different diagrams is

$$\mathcal{M}=-6\lambda\left[1+3(m_\sigma^2-m_\pi^2)\times\left(\frac{1}{s-m_\sigma^2}+\frac{1}{t-m_\sigma^2}+\frac{1}{u-m_\sigma^2}\right)\right]. \tag{54}$$

This reflects the symmetry in the s, t, and u channels, and $s+t+u=4m_\sigma^2$. The total cross section is

$$\sigma_{\sigma\sigma}(s)=\frac{9\lambda^2}{8\pi s}\left[\left(\frac{s+2m_\sigma^2-3m_\pi^2}{s-m_\sigma^2}\right)^2+\frac{18(m_\sigma^2-m_\pi^2)^2}{m_\sigma^2(s-3m_\sigma^2)}-\frac{12(m_\sigma^2-m_\pi^2)(s^2-3sm_\sigma^2-m_\sigma^4+3m_\sigma^2m_\pi^2)}{(s-4m_\sigma^2)(s-2m_\sigma^2)(s-m_\sigma^2)}\times\ln\left(\frac{s-3m_\sigma^2}{m_\sigma^2}\right)\right]. \tag{55}$$

In the low energy limit expand this about $s=4m_\sigma^2$. To leading order

$$\sigma_{\sigma\sigma}=\frac{9}{32\pi}\lambda^2\frac{(4m_\sigma^2-5m_\pi^2)^2}{m_\sigma^6}, \tag{56}$$

and in the rest-frame of the sigma this gives rise to

$$\mathrm{Im}\,\Pi_{\sigma\sigma}=-\frac{9}{32\pi^3}\lambda^2T^2e^{-m_\sigma/T}\times\frac{(m_\sigma+T)(4m_\sigma^2-5m_\pi^2)^2}{m_\sigma^5}. \tag{57}$$

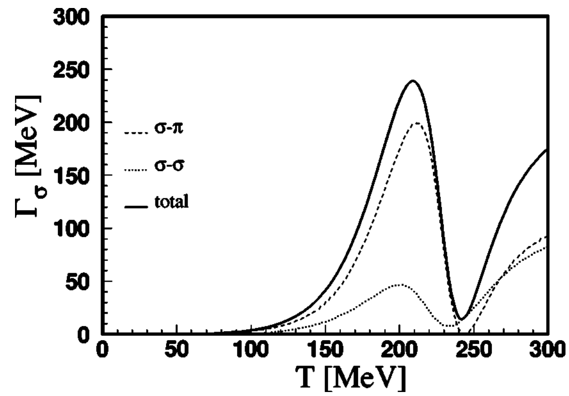

FIG. 9. Scattering contribution to the width of the sigma meson as a function of temperature. Calculations were done with resummed meson masses.

In the high energy limit only the four-point vertex contributes to the amplitude, resulting in

$$\sigma_{\sigma\sigma}=\frac{9\lambda^2}{8\pi s}. \tag{58}$$

This gives rise to

$$\mathrm{Im}\,\Pi_{\sigma\sigma}=-\frac{3}{64\pi}\lambda^2T^2. \tag{59}$$

With these two limits I construct an interpolating formula that describes the whole energy range. The contribution to the rate of decay of the amplitude is then

$$\Gamma_{\sigma\sigma}\simeq\frac{9\lambda^2}{64\pi}\times\frac{T^2(m_\sigma+T)(4m_\sigma^2-5m_\pi^2)^2}{6m_\sigma(m_\sigma+T)(4m_\sigma^2-5m_\pi^2)^2+\pi^2m_\sigma^6(e^{m_\sigma/T}-1)}. \tag{60}$$

A hard pion in the heat bath can be energetic enough to knock out a sigma meson from the condensate. The possible tree-level processes may happen according to four different diagrams. The transition amplitude obtained from these is

$$\mathcal{M}=-2\lambda\left[1+(m_\sigma^2-m_\pi^2)\times\left(\frac{1}{s-m_\pi^2}+\frac{3}{t-m_\sigma^2}+\frac{1}{u-m_\pi^2}\right)\right], \tag{61}$$

where $s+t+u=2(m_\sigma^2+m_\pi^2)$. The total scattering cross section is

$$\sigma_{\sigma\pi}(s)=\frac{\lambda^2}{4\pi s}\Bigg[\left(\frac{s-2m_\pi^2+m_\sigma^2}{s-m_\pi^2}\right)^2+\frac{9s(m_\sigma^2-m_\pi^2)^2}{m_\sigma^2[s^2-sm_\sigma^2-2sm_\pi^2+(m_\sigma^2-m_\pi^2)^2]}+\frac{s(m_\sigma^2-m_\pi^2)^2}{(2m_\sigma^2+m_\pi^2-s)[(m_\sigma^2-m_\pi^2)^2-sm_\pi^2]}$$
$$+\frac{6s(m_\sigma^2-m_\pi^2)(sm_\sigma^2+2sm_\pi^2-s^2+m_\sigma^4-m_\pi^4-2m_\sigma^2m_\pi^2)}{(s-m_\pi^2)(m_\sigma^2+m_\pi^2-s)[s^2-2s(m_\sigma^2+m_\pi^2)+(m_\sigma^2-m_\pi^2)^2]}\ln\frac{sm_\sigma^2}{s^2-sm_\sigma^2-2sm_\pi^2+(m_\sigma^2-m_\pi^2)^2}$$
$$-\frac{2s(m_\sigma^2-m_\pi^2)(3sm_\sigma^2-s^2+m_\pi^4+m_\sigma^4-4m_\sigma^2m_\pi^2)}{(s-m_\pi^2)(m_\sigma^2+m_\pi^2-s)[s^2-2s(m_\sigma^2+m_\pi^2)+(m_\sigma^2-m_\pi^2)^2]}\ln\frac{s(2m_\sigma^2+m_\pi^2-s)}{(m_\sigma^2-m_\pi^2)^2-sm_\pi^2}\Bigg]. \quad (62)$$

The low energy limit is obtained by expanding this about $s=(m_\sigma+m_\pi)^2$ and is

$$\sigma_{\sigma\pi}(s)=\frac{9}{4\pi}\lambda^2\frac{m_\pi^4(3m_\sigma^2-4m_\pi^2)^2}{sm_\sigma^2(m_\sigma^2-4m_\pi^2)^2}. \quad (63)$$

At high temperatures, where only the four-vertex diagram contributes, the cross section is reduced to

$$\sigma_{\sigma\pi}(s)=\frac{\lambda^2}{4\pi s}. \quad (64)$$

The contribution to the imaginary part of the self-energy at low energies is

$$\mathrm{Im}\,\Pi_{\sigma\pi}=-\frac{9}{4\pi^3}\lambda^2T^2e^{-m_\pi/T}F_{\sigma\pi}(m_\sigma,m_\pi), \quad (65)$$

where I defined

$$F_{\sigma\pi}(m_\sigma,m_\pi)=\frac{m_\pi^4(m_\pi+T)(3m_\sigma^2-4m_\pi^2)^2}{m_\sigma^3(m_\sigma+m_\pi)^2(m_\sigma^2-4m_\pi^2)^2}, \quad (66)$$

and at high energies

$$\mathrm{Im}\,\Pi_{\sigma\pi}=-\frac{\lambda^2T^2}{96\pi}. \quad (67)$$

The two limits can be combined into one approximate expression which then determines the rate of dissipation. Factoring in all the $N-1$ pions

$$\Gamma_{\sigma\pi}\simeq\frac{9(N-1)}{8\pi}\lambda^2\frac{T^2}{m_\sigma}\frac{F_{\sigma\pi}}{216F_{\sigma\pi}+\pi^2(e^{m_\pi/T}-1)}. \quad (68)$$

A distinction should be made between scatterings with massless and massive pions. The low energy expression (63) vanishes for zero pion mass. The first nonzero term in the series expansion of the cross section is the fourth order term. I discuss the massless case in my forthcoming paper [22].

The temperature dependence of the total scattering rate of the sigma meson, $\Gamma_{\sigma\sigma}+\Gamma_{\sigma\pi}$, evaluated with the self-consistently determined meson masses is shown in Fig. 9. Scattering is more accentuated at higher temperatures and its contribution to the sigma damping rates is well below the energy. Dissipation of sigmas from the condensate due to their scattering is much smaller than due to their decay, meaning that the scalar order parameter relaxes to its equilibrium value via the production of lighter pion fields. It also means that sigma mesons are so unstable that they are more likely to decay before they could ever scatter with other particles from the medium.

2. Pion scattering

Dissipation of DCCs can arise from scattering of the soft pions with hard pions or hard sigma mesons. The damping of massive pions and that of the Goldstone pions I expect to be different, requiring a somewhat different analysis. In the following I present the discussion on massive pions. There are four possible tree-level diagrams representing the reaction in which a hard thermal sigma knocks out a low momentum pion from the condensate. The total cross section is the same as for sigma-pion scattering and is given by expression (62). The imaginary part of the self-energy in the low energy limit in the rest frame of the pion is

$$\mathrm{Im}\,\Pi_{\pi\sigma}=-\frac{9}{4\pi^3}\lambda^2T^2e^{-m_\sigma/T}F_{\pi\sigma}, \quad (69)$$

where

$$F_{\pi\sigma}=\frac{m_\pi^5(m_\sigma+T)(3m_\sigma^2-4m_\pi^2)^2}{m_\sigma^4(m_\sigma+m_\pi)^2(m_\sigma^2-4m_\pi^2)^2}, \quad (70)$$

and for high energies is

$$\mathrm{Im}\,\Pi_{\pi\sigma}=-\frac{\lambda^2T^2}{96\pi}. \quad (71)$$

The total scattering rate due to this process can be parametrized by an interpolating formula between the two known limits:

$$\Gamma_{\pi\sigma}\simeq\frac{9\lambda^2T^2}{8\pi m_\sigma}\frac{F_{\pi\sigma}}{216F_{\pi\sigma}+\pi^2(e^{m_\sigma/T}-1)}. \quad (72)$$

$\pi\pi$ scattering has been extensively studied during the last couple of decades in a variety of different models and approaches. An incomplete but significant list of references is [37]. Here I study the elastic scattering of a hard pion off a soft pion. For one pion species, $N=2$, there are four diagrams contributing to this process at tree-level: one 4-point vertex diagram and three 3-point vertex contributions involving a sigma exchange in the s, t, and u channels. The transition amplitude is

$$\mathcal{M}=-2\lambda\left[3+(m_\sigma^2-m_\pi^2)\times\left(\frac{1}{s-m_\sigma^2}+\frac{1}{t-m_\sigma^2}+\frac{1}{u-m_\sigma^2}\right)\right], \tag{73}$$

where $s+t+u=4m_\pi^2$. Accounting for all pion species opens up additional channels. For $N=4$ the transition amplitude averaged over initial and summed over final isospins is

$$|\mathcal{M}|^2=\frac{1}{3}\sum_{I=0,1,2}(2I+1)|\mathcal{M}^I|^2, \tag{74}$$

where $\mathcal{M}^{\mathcal{I}}$ is the matrix element associated with the total isospin I of the two-pion system:

$$\mathcal{M}^0=-2\lambda\left[5+(m_\sigma^2-m_\pi^2)\times\left(\frac{3}{s-m_\sigma^2}+\frac{1}{t-m_\sigma^2}+\frac{1}{u-m_\sigma^2}\right)\right],$$

$$\mathcal{M}^1=-2\lambda(m_\sigma^2-m_\pi^2)\left(\frac{1}{t-m_\sigma^2}-\frac{1}{u-m_\sigma^2}\right),$$

$$\mathcal{M}^2=-2\lambda\left[2+(m_\sigma^2-m_\pi^2)\left(\frac{1}{t-m_\sigma^2}+\frac{1}{u-m_\sigma^2}\right)\right]. \tag{75}$$

The resulting cross section is

$$\sigma_{\pi\pi}(s)=\frac{\lambda^2}{8\pi s}\left[15+10\left(\frac{m_\sigma^2-m_\pi^2}{s-m_\sigma^2}\right)+3\left(\frac{m_\sigma^2-m_\pi^2}{s-m_\sigma^2}\right)^2+\frac{6(m_\sigma^2-m_\pi^2)^2}{m_\sigma^2(s+m_\sigma^2-4m_\pi^2)}\right.$$
$$\left.-\frac{4(m_\sigma^2-m_\pi^2)(5s^2+5sm_\sigma^2-7m_\sigma^4+13m_\sigma^2m_\pi^2-20sm_\pi^2+4m_\pi^4)}{(s-4m_\pi^2)(s-m_\sigma^2)(s+2m_\sigma^2-4m_\pi^2)}\ln\left(\frac{s+m_\sigma^2-4m_\pi^2}{m_\sigma^2}\right)\right]. \tag{76}$$

The low-energy limit of the cross section, quite acceptable for $m_\sigma \gg T$, is given by the expansion of this about $s=4m_\pi^2$,

$$\sigma_{\pi\pi}=\frac{\lambda^2}{32\pi}\frac{m_\pi^2}{(m_\sigma^2-4m_\pi^2)^2}\left(23-16\frac{m_\pi^2}{m_\sigma^2}+128\frac{m_\pi^4}{m_\sigma^4}\right). \tag{77}$$

At high temperatures, in the $T\rightarrow T_c$ limit the major contribution to the amplitude is from the four-point vertices, resulting in

$$\sigma_{\pi\pi}=\frac{15\lambda^2}{8\pi s}. \tag{78}$$

In the rest frame of the pion the above two limits give

$$\mathrm{Im}\,\Pi_{\pi\pi}\simeq-\frac{23}{32\pi^3}\lambda^2T^2e^{-m_\pi/T}\frac{m_\pi^3(m_\pi+T)}{(m_\sigma^2-4m_\pi^2)^2} \tag{79}$$

and

$$\mathrm{Im}\,\Pi_{\pi\pi}=-\frac{5\lambda^2T^2}{64\pi}, \tag{80}$$

respectively. The rate of dissipation due to massive pion-pion scattering is then given by the interpolating expression

$$\Gamma_{\pi\pi}\simeq\frac{23\lambda^2T^2}{64\pi}\times\frac{m_\pi^2(m_\pi+T)}{\frac{46}{5}m_\pi^3(m_\pi+T)+\pi^2(m_\sigma^2-4m_\pi^2)^2(e^{m_\pi/T}-1)}. \tag{81}$$

The scattering contribution to the damping of massive pions is presented in Fig. 10. Because of the heavy sigma exchange there is a strong suppression at low temperatures. When reaching $T\simeq130$ MeV the scattering rate becomes

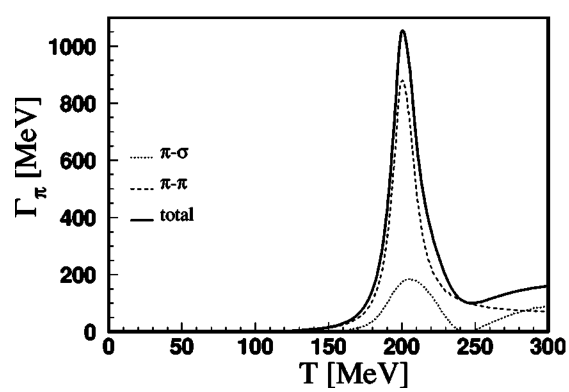

FIG. 10. Scattering contribution to the width of the pion as a function of temperature. Calculations were done with resummed meson masses.

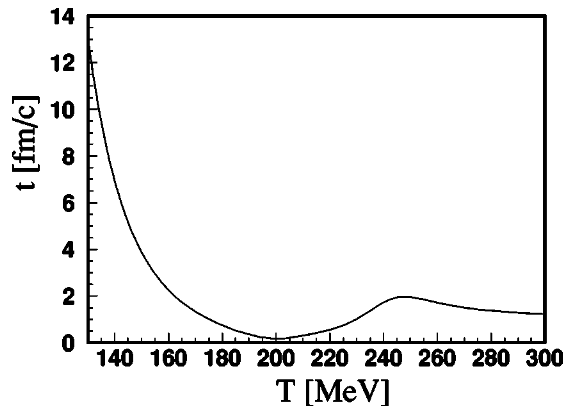

FIG. 11. Relaxation time of a homogeneous disoriented chiral condensate versus temperature.

significant and it is comparable in magnitude to the damping at one-loop order. In the critical region the contribution to the pion width from pion-pion scattering grows rapidly reaching a maximum about $T=200$ MeV. It is interesting to note from Figs. 9 and 10 that in the high temperature regions, where the mesons become almost degenerate, their widths approach the same value, as expected.

VI. RELAXATION TIME

The ability to detect DCCs in relativistic collisions of heavy ions depends on the lifetime of the condensate. DCC formation can happen out of thermal equilibrium only. In order to talk about nonequilibrium physics the rate of expansion t_{exp}, has to be much smaller than the relaxation time of long-wavelength modes, t,

$$t_{exp} \ll t. \tag{82}$$

Otherwise, for a slow expansion, the soft modes have enough time to equilibrate. Equilibration of the out-of-equilibrium chiral condensate is the result of the presence of a heat bath. Above, I analyzed the physical processes responsible and calculated the damping of different meson modes. The total width is the sum of one- and two-loop order dissipative contributions,

$$\Gamma_\pi = \Gamma_{\pi\pi\sigma} + \Gamma_{\pi\sigma} + \Gamma_{\pi\pi}, \tag{83}$$

which exhibits a sharp peak in the critical region, due to the peak in the damping rate from pion-pion scattering. In the nonequilibrium, but close to equilibrium physics that we consider, the damping directly controls the rate at which equilibrium is approached through the relation [34]

$$t = \frac{1}{\Gamma_\pi}. \tag{84}$$

Obviously, the larger the damping due to the interaction of the condensate with the heat bath, the shorter the relaxation time is. Figure 11 shows the change in the relaxation time with temperature. At low temperatures t is large due to the suppression of the thermal occupation numbers. As the temperature increases more thermal modes get excited and as a consequence the relaxation time is quickly decreasing. In the phase transition region the decay time is the shortest. At the peak of the damping $T=200$ MeV I found $t=0.17$ fm/c, and at $T=235$ MeV, where the mass of the sigma is the smallest, $t=1.36$ fm/c. When assuming no multiple interactions with the heat bath then t is the lifetime of the DCC. The times I obtained are shorter than previous estimates [9,18,19], and are short enough to make a possible DCC signal questionable. One can expect that any multiple scatterings or decays would only increase the damping, decreasing the relaxation time.

VII. CONCLUSIONS

This work was motivated by my interest to determine the possibility of survival of DCCs in the background of a multitude of thermal particles, mostly pions, that are formed after two heavy ions are collided at ultrarelativistic energies. Also, having a consistent description of quantum fields near thermal equilibrium allows for a better understanding of their dynamics in a region where nonperturbative analysis is required.

We have developed a consistent semiclassical study of the out-of-equilibrium chiral condensate fields in the framework of the linear sigma model. Clear distinction between the soft nonthermal chiral fields and hard thermal modes has been made, accounting also for interactions between these. Motivated by the high occupancy of the low momentum modes I allowed for their classical treatment. The effect of the other degrees of freedom has been taken into account by introducing a heat bath of mesons. These thermalized high momentum modes have been accounted for in a perturbative manner, improved by the resummation of certain diagrams. I derived classical equations of motion for the long-wavelength condensate fields coupled to the thermal bath. After integrating out the hard modes effective field equations resulted, which completely determine the evolution of the chiral condensate in space and time.

The presence of the slowly varying condensate fields cause deviations in the equilibrium fluctuations of the thermal fields. I identified these as linear response functions, since I am dealing with not too far from equilibrium scenarios. I have discussed in detail the richness of information contained within these response functions: They renormalize the equations of motion, modifying the particle properties, and give rise to dissipation.

The temperature dependence of the meson masses and that of the equilibrium condensate have been determined numerically in a self-consistent manner, both in the chiral limit and for explicitly broken chiral symmetry. In the chiral limit a first-order phase transition was found, which is an artifact of the model. I show that for a small coupling constant, or large N limit, the expected second-order transition is recovered. I have considered in some detail the Goldstone boson nature of the pion, proving that when properly accounting for the tadpole and sunset diagrams the pion remains massless at one-loop level in the symmetry broken phase. Also, the tachyon problem of mean-field approximations is eliminated by naturally assuring the positivity of the masses at all temperatures. In the more realistic case, in which chiral symmetry is explicitly broken by the nonzero quark mass, a crossover region was identified. The minimum of the sigma mass is at the transition temperature of about 235 MeV. Above this the masses of the pion and the sigma become degenerate and the equilibrium condensate vanishes asymptotically, signaling an approximate restoration of chiral symmetry.

Because of possible interactions between different degrees of freedom, those of the condensate and those of the heat bath, energy exchange is possible and particles can be knocked out or put in the condensate. Direct evaluation of the response functions results in the rates for the different processes. I have identified these physical processes that are responsible for the dissipation of long-wavelength modes of the chiral condensate, and have confirmed that at high temperatures not only the damping of the sigmas is significant, but also that of the pions. At one-loop level, provided a kinematic condition is satisfied, a pion from the condensate can annihilate with a pion from the heat bath forming a sigma. The damping due to this process becomes stronger with increasing temperature. The width of the pion can be as big as 55% of its energy. This result makes me question whether one can talk about the pion as a quasiparticle in the phase transition region.

I emphasize the importance of two-loop calculations. My results show that contribution to the damping rate of the condensate provided by two-particle elastic scattering processes can become as important as are decay processes. Moreover, while decays happen only when certain kinematic conditions are satisfied, elastic scatterings have no such restrictions. Therefore two-loop processes contribute to the width even in regions where one-loop contributions are suppressed. Soft pions from the condensate can be knocked out through elastic scatterings with a hard thermal pion or sigma. The damping due to these is most accentuated in the phase transition region. The pion width shows a peak at about 200 MeV.

The damping directly controls the rate at which equilibrium is achieved by the nonequilibrium condensate. I have determined the relaxation time of a homogeneous condensate due to both one- and two-loop order dissipative processes. We have obtained relaxation times between 0.17 and 1.36 fm/c in the phase transition region. Assuming no multiple interactions these times become the lifetime of the condensate. I have found that the lifetime of disoriented chiral condensates is short enough to make a possible DCC signal questionable. Further investigations of this model in a far from equilibrium scenario are needed.

The natural next step of my investigation is to solve the field equations for some initial conditions for an expanding system. Such analysis is on the way.

The methods used in this paper are general, and may be used in other contexts, where nonequilibrium physics of quantum fields is of interest.

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